WASHING OF PULP FIBRE BED

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The mechanism of the displacement washing of unbleached kraft pulp was investigated. The flow of the wash liquid through a pulp bed was described by the dispersion model using one dimensionless parameter known as the Peclet number. To characterize the displacement washing, the wash yield, dispersion coefficient, and bed efficiency, as well as time parameters such as the mean residence time of lignin, and space time were evaluated. Results obtained for kraft pulp were compared with those for static bed of glass spheres.
Key words: Displacement washing; Pulp; Dispersion model.

The purpose of pulp washing is to remove the spent pulping liquor from the pulp leaving the cooking process. Water added to the liquor during washing must be removed again by evaporation. Hence, the objective of pulp washing is not only to remove as much of the soluble impurities as possible, but to do it with as little amount of water as possible. The washing process must compromise between the cleanliness of the pulp and the amount of water to be used[1].

Two basic pulp washing operations, namely dilution-thickening and displacement washing, are used industrially. In dilution-thickening washing, the pulp slurry is diluted and thoroughly mixed with weak wash liquor or clean water and then thickened by filtering or by pressing. In displacement washing, wash liquor or clean water passes through a pulp bed in a piston-like manner, pushing out the liquor which had been associated with the pulp. In comparison with dilution-thickening washing, the displacement washing is more effective if the amount of wash liquid added to the pulp is the same. Ideally, it would be possible to displace one volume of the liquor in the pulp bed with the same volume of clean wash water. In practice, however, all of the soluble impurities cannot be washed out. Because of the structural anisotropies, the flow of wash water through a pulp bed is different from the perfect plug flow.

Several papers[2–5] were devoted to the displacement washing process. The effect of pulp bed consistency (the consistency is the concentration of moisture-free pulp fibres expressed in mass % or kg m^{-3}), pulp bed thickness, wash liquid velocity, and temperature of wash liquid on displacement washing efficiency was investigated. Because of the complexity of the relationships between these process variables and the washing efficiency as well as the use of different kinds of pulp and experimental techniques, the results so far reported disagree with each other.

The aim of this work is to describe the displacement washing process of fibre pulp from the point of view of chemical-engineering analysis. The purpose of our investigation was to determine the wash yield, efficiency of pulp bed with respect to lignin removal out of the pulp, as well as the effect of diffusion process on displacement washing. As the particles in the pulp bed are porous, the flow of wash water through the bed of non-porous glass spheres was also investigated so that both the systems might be compared.

THEORETICAL

Step function input signal has been widely used in the analysis of the flow through a packed bed of solid particles[6]. A response to a step change in concentration is known as the washing curve which plots the ratio of the instantaneous outlet to the initial mother liquor concentration in the bed against the quantity of the wash liquid used. The quantity of the wash liquid is usually expressed as the wash liquor ratio, *RW* (ref.[7]), which is defined as the ratio of the amount of wash liquid passed through the bed and the initial amount of mother liquor present in the bed. The displacement washing curve area is directly proportional to the amount of solute (*e.g.* organic compounds or sodium, in our case lignin) removed.

Quality of the displacement washing can be characterized by the wash yield. The displacement wash yield, *WY* (see ref.[1]), is defined as the amount of lignin washed out at some wash liquor ratio divided by the total amount of lignin removed during the washing period. The displacement wash yield, $WY_{RW=1}$, evaluated at the wash liquor ratio equal to unity is often used as a standard basis of comparison[5]. This yield may be expressed as

$$WY_{RW=1} = \frac{\int\limits_{RW=0}^{RW=1} \frac{c_e}{c_0}\, \mathrm{d}(RW)}{\int\limits_{RW=0}^{RW\to\infty} \frac{c_e}{c_0}\, \mathrm{d}(RW)} \; . \qquad (1)$$

The equation for predicting the displacement wash yield in any particular case should include all the properties of the pulp bed and wash liquid, and the conditions of its flow that affect the washing process. In particular case these factors might be the interstitial velocity of the liquid flowing through the bed, the dispersion of wash liquid inside the bed, as well as the physical properties of the bed such as its porosity, its permeability, and the specific surface of fibres together with the intrinsic properties of the wash liquid, namely its density and its viscosity, and possibly others. The extent of process variables (pulp bed consistency, bed thickness, temperature, and displacement velocity) is so wide that it is practically hopeless to assemble all factors into an equation that shall be based on purely theoretical reasoning.

The flow through a packed bed is often described by the dispersed plug flow model. For the one-dimensional case, when only axial dispersion is considered, the basic differential equation can be written in the form

$$\frac{\partial c_i}{\partial t} = D \frac{\partial^2 c_i}{\partial z^2} - \frac{u}{\varepsilon} \frac{\partial c_i}{\partial z} \quad . \tag{2}$$

The boundary conditions appropriate to the displacement process have been extensively discussed by Brenner[8]. At the inlet to the bed, where $z = 0$, the boundary condition is

$$\frac{u}{\varepsilon} c_i - D \frac{\partial c_i}{\partial z} = \frac{u}{\varepsilon} c_v \quad \text{for all } t > 0 \quad . \tag{3}$$

This condition is imposed by the requirement that there be no loss of solute from the bed through the plane at which the displacing liquid is introduced. In order to avoid the unacceptable conclusion that the solute concentration passes through a maximum or minimum in the interior of the bed, it is necessary to impose the second boundary condition

$$\frac{\partial c_i}{\partial z} = 0 \quad \text{at } z = h \text{ for all } t > 0 \tag{4}$$

for the bed exit. The initial condition is

$$c_i(z,0) = c_0 \quad \text{for all } z \quad . \tag{5}$$

Introducing the dimensionless concentration $C_i = (c_i - c_v)/(c_0 - c_v)$, dimensionless length coordinate $Z = z/h$, and dimensionless time, T, defined as

$$T = \frac{ut}{\varepsilon h} \quad , \tag{6}$$

Eq. (*3*) can be put in the following dimensionless form

$$\frac{\partial C_i}{\partial T} = \frac{1}{Pe} \frac{\partial^2 C_i}{\partial Z^2} - \frac{\partial C_i}{\partial Z} , \tag{7}$$

where

$$Pe = \frac{hu}{D\varepsilon} \tag{8}$$

is the Peclet number indicating the level of dispersion in the bed. Physically this means that, when the Peclet number approaches zero, the bed behaves like a perfect mixing vessel. On the other hand, the perfect plug flow is characterized by $Pe \to \infty$.

The asymptotic solution of Eq. (*7*) along with two step function boundary and one initial condition rewritten in dimensionless form was given by Brenner[8]. The solution of the axially dispersion model of miscible fluid displacement in beds of finite thickness was expressed in the dependence of the exit concentration, c_e, upon the dimensionless time, T, for several values of the Peclet number. The dimensionless time, T, defined by Eq. (*6*) is equal to the ratio of total fluid volume introduced into the free volume of the bed.

EXPERIMENTAL

The experiments were carried out in the displacement washing cell consisting of a vertical glass cylinder 110 mm high, 36.4 mm inside diameter, and closed at the lower end by a permeable septum. The fibre bed occupied the volume between the septum and a permeable piston which could be slid into the top of the cylinder. Both the piston and the septum were made permeable by 64 holes of 1 mm diameter. The ratio of the hole area to the cylinder cross-sectional area was then 4.9%. This apparatus was very similar to that used by Lee[3]. Of course, the inside diameter of the washing cell used by Lee[3] was 75 mm.

Pulp beds were formed from a dilute suspension of unbeaten unbleached kraft pulp in black liquor. In order to characterize the pulp fibres used in experiments, physical properties of kraft pulp were determined as well. The Schopper–Riegler freeness had a value of 13 deg SR. The freeness of pulp was designed[9] to give a measure of the rate at which a dilute suspension of pulp may be drained. The results obtained by freeness test are used to assess the degree of beating of pulp given. The degree of delignification of pulp was expressed in terms of the kappa number (see Table I). The kappa number is the volume (in milliliters) of 0.1 mol l^{-1} potassium permanganate solution consumed by one gram of moisture-free pulp under the conditions specified in test method[10]. Using the Kajaani instrument[11], the mean length of the fibres was also determined (Table I). In all runs, the beds were compressed to a final desired thickness of 30 mm. Thus, the length ratio of the bed thickness to the inner washing cell diameter, h/d, had always a value of 0.82. The pulp beds were not mechanically conditioned and were used as formed. The consistency of the oven dried pulp in bed varied in the range from 12.1 to 14.8 mass %.

Most runs were made with wood pulp. A few runs were performed with non-porous glass particles. The washing cell was randomly packed with 1.9 mm spherical particles to a height of 30 mm that was kept constant in all experiments.

To start the washing experiment, distilled water maintained at a temperature of 20–22 °C was distributed uniformly through the piston to the top of bed, approximating a step change in concentration. At the same time, the displaced liquor was collected at atmospheric pressure from the bottom of the bed *via* the septum. Depending on the permeability of the pulp bed formed in washing cell, the wash water was forced through the pulp bed under pressure drop up to 7 kPa.

Samples of the washing effluent taking at different time intervals until the effluent was colourless were analysed for lignin using an ultraviolet spectrophotometer operating at a wavelength of 295 nm. After finishing the washing run, the pulp bed was removed from the apparatus, weighed, and oven dried at 105 °C.

Four series of experiments were performed. Table II shows the range of experimental conditions. The concentration of total solids in the black liquor from which the pulp slurry was prepared varied from 16.7 to 18.8 mass %, the range of pH value was 12.8–13.0, and density of black liquor was between 1 083 and 1 094 kg m^{-3}.

After completing the washing experiment, the volumetric wash liquid flow rate was measured gravimetrically at various values of bed consistency in order to determine the properties of the pulp bed such as its permeability, effective specific volume of fibres, and specific surface of fibres.

Treatment of Experimental Data

In order to characterize the displacement washing curves measured for lignin, the Peclet number was evaluated. For pulp fibres used in our experiments, properties of swollen fibres were determined experimentally. Also, a relationship for bed efficiency expressing lignin removal during washing process was modified for our experimental conditions.

TABLE I
Physical properties of pulp fibres

Set of runs	Kappa number	l_n, mm	l_w, mm	$v \cdot 10^3$, m^3 kg^{-1}	$a \cdot 10^{-5}$, m^{-1}
I, II	17.6	1.56	2.47	2.49	3.33
III	21.5	1.09	2.01	2.88	2.43

TABLE II
Experimental conditions

Set of runs	Number of measurements	Particles	Temperature of wash liquid, °C	c_0, kg m^{-3}	$u \cdot 10^5$, m s^{-1}	*Pe*
I	12	fibres	20	29	4.0–21	2.2–20
II	8	fibres	22	42	6.3–19	4.1–12
III	8	fibres	20	53	2.2–29	1.0–4.5
IV	8	spheres	21	42	8.9–32	33–82

Peclet Number

The first step in the data treatment consisted in converting the dependence of lignin concentration in exit stream, c_e, on time, t, to a plot of dimensionless concentration, c_e/c_0, *versus* the dimensionless time, Θ, expressed as

$$\Theta = \frac{t}{t_m} \quad , \tag{9}$$

where the mean residence time t_m is given as

$$t_m = \int_0^\infty \frac{c_e}{c_0}\, dt \quad . \tag{10}$$

If the normalized displacement washing curve is obtained, one may reduce it by differentiation to the corresponding response to the pulse input signal which can be characterized by its variance σ^2 (refs[12,13]) defined as

$$\sigma^2 = \int_0^\infty (\Theta - 1)^2 E_c \, d\Theta \quad , \tag{11}$$

where

$$E_c = -\frac{d(c_e/c_0)}{d\Theta} \tag{12}$$

is a measure of the age distribution of lignin leaving a bed. The precise relationship between the variance and the Peclet number depends on the end conditions. For a closed vessel of finite length, where no solute (in our case lignin) moves into and out of the vessel by dispersion, the relationship between the variance of the distribution of residence time and the Peclet number was derived by Laan[14] in the form

$$\sigma^2 = \frac{2}{Pe} - \frac{2}{Pe^2}\,[1 - \exp(-Pe)] \quad . \tag{13}$$

From the Peclet number, the influence of the dispersion on the displacement process can then be predicted.

Properties of Pulp Bed

The volumetric liquid flow rate through pulp bed is given by Darcy's law in the form

$$V = B\,\frac{A\,\Delta p}{\mu h} \tag{14}$$

which holds in streamline flow regime. The permeability of the pulp bed can be expressed as

$$B = R^2 \frac{\varepsilon}{K} \ , \tag{15}$$

where the hydraulic mean radius of the pores is given as

$$R = \frac{\varepsilon}{(1 - \varepsilon)a} \ . \tag{16}$$

Substituting Eq. (*16*) into Eq. (*15*), the relationship for the permeability of the pulp can be obtained as

$$B = \frac{\varepsilon^3}{(1 - \varepsilon)^2 a^2 K} \ , \tag{17}$$

where the factor, K, is called the Kozeny constant and is dependent only upon the shape of pores and the ratio of the tortuous length that liquid traverses in passing through the bed to the actual thickness of the bed. It was found that the Kozeny constant has an average value of 5.55 for randomly packed fibrous beds and shows no apparent variation with porosity, ε, within the range of 0.45 to 0.86 (ref.[15]).

According to Ingmanson[15], the porosity, ε, can be expressed in terms of the consistency of fibres in pulp bed, C, and the effective specific volume of the swollen fibres, v. Thus,

$$\varepsilon = 1 - vC \ . \tag{18}$$

It must nevertheless be stressed that the effective specific volume involves not only the volume of the fibres in the water-swollen state but also the volume of the liquid immobilized on their surface.

After substituting Eq. (*18*) into Eq. (*17*) and subsequent rearranging, we obtain the folowing equation

$$(BC^2)^{1/3} = (v^2 a^2 K)^{-1/3} - (v^2 a^2 K)^{-1/3} vC \ . \tag{19}$$

The effective specific volume, v, and the specific surface of the fibres, a, can be determined by a treatment of permeability data obtained at various values of bed consistency. Detailed description of this method is given in the paper[15].

Table I shows the values of the effective specific volume and the specific surface of pulps used in our work. Ingmanson[15] found that the effective specific volume of unbeaten bleached sulfite pulp had a value of $2.21 \cdot 10^{-3}$ m^3 kg^{-1}. For kraft cooked blend of hardwood and softwood, Lee[3] reported the effective specific volume of $3.33 \cdot 10^{-3}$ m^3 kg^{-1}. Our values of the effective specific volume (see Table I) lie between the values published by Ingmanson[15] and Lee[3]. It can be assumed that the effective specific volume depends on the length of fibres. It is known that the fibre length of hardwood have less value in comparison with the fibre length of softwood.

Bed Efficiency

The efficiency of a packed bed may be also defined by the relationship similar to the Murphree efficiency, which is often used for distillation trays, as

$$E = \frac{c_e - c_V}{c_L - c_V} \ , \tag{20}$$

where c_e is the lignin concentration in the outlet stream, c_V is the lignin concentration in the wash liquor, and c_L is the average lignin concentration inside the bed. The dependence of the average solute (in our case lignin) concentration inside packed beds of finite thickness on the dimensionless time, *T*, is reported by Brenner[8].

For displacement washing, Cullinan[16] derived the relationship between the efficiency of a pulp bed, *E*, the wash liquor ratio, *RW*, and the displacement ratio, *DR*, in the form

$$DR = 1 - \exp\,(-E\,RW) \ . \tag{21}$$

In our case, the displacement ratio may be defined as

$$DR = \frac{c_0 - c_L(RW)}{c_0 - c_V} \ , \tag{22}$$

where c_0 denotes the initial lignin concentration inside the pulp bed at $RW = 0$ and $c_L(RW)$ denotes the average lignin concentration inside the pulp bed at $RW = RW$. If the lignin concentration in the wash liquor $c_V = 0$ and the initial and final consistencies are the same, that is, the liquor volume associated in the pulp bed has a constant value, the displacement ratio at $RW = 1$ is equal to the wash yield defined by Eq. (*1*). Then, the bed efficiency can be expressed as

$$E = -\frac{\ln\,(1 - WY)}{RW} \ . \tag{23}$$

The bed efficiency evaluated at the wash liquor ratio equal to unity, $E_{RW=1}$, was used to compare the lignin removal during the displacement washing process.

RESULTS AND DISCUSSION

Wash Yield

Typical displacement washing curves are presented in Fig. 1. For a better optical comparison of washing curves, the experimental points were connected by means of the cubic spline method. The shape of the curves obtained experimentally indicates that the flow through the pulp bed during displacement washing was non-ideal, *i.e.* it is between the ideal limits of plug flow and perfectly mixed flow. It must be noted that the washing experiments were finished at the wash liquor ratio equal to about 7 when the lignin concentration in output stream was less than one thousandth of the initial lignin concentration in the pulp bed. Owing to the nature of the system studied, the perfect plug flow displacement cannot be achieved for the dispersion due to the inhomogeneities of the fibre bed, *e.g.* stagnant liquor regions, by-pass flows, and the like, had a very significant effect on the flow of wash liquid through the pulp bed. In our case, the pulp bed can be characterized as a fixed bed consisted of compressible porous particles where geometrical similarity does not exist. In contrast with pulp bed, the shape of washing curves obtained for the glass sphere bed approaches to that for plug flow. At the wash liquor ratio equal to approximately 2.5, all lignin has already been removed from the glass sphere bed.

In Fig. 2, the wash yield, $WY_{RW=1}$, evaluated from Eq. (*1*) is plotted against the Peclet number. The experimental points are located along the curve which was generated using the dependence of the wash yield, $WY_{T=1}$, defined as

$$WY_{T=1} = \frac{\int_{T=0}^{T=1} \frac{c_e}{c_0}\, dT}{\int_{T=0}^{T\to\infty} \frac{c_e}{c_0}\, dT} \qquad (24)$$

on the Peclet number. The wash yield, $WY_{T=1}$, was determined by generating the dependence of the dimensionless exit concentration, c_e/c_0, upon the dimensionless time, T, for various values of the Peclet number using the tabulation of Brenner[8], then calculating the wash yield, $WY_{T=1}$, for each theoretical washing curve.

The wash yield, $WY_{RW=1}$, obtained experimentally depends not only upon the hydrodynamics of washing but also on the initial bed lignin concentration in black liquor. Our own experimental results showed that the wash yield decreases with increasing initial bed lignin concentration. Thus, three sets of experiments are represented by three

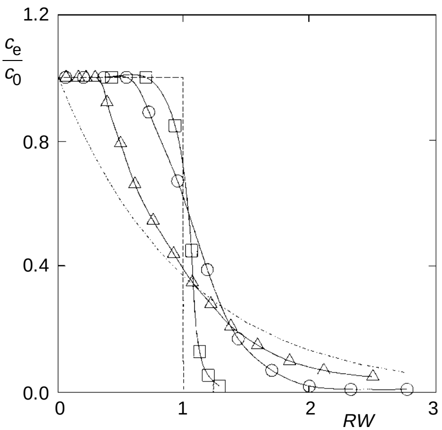

FIG. 1
Typical displacement washing curves: Δ kraft pulp (kappa number = 21.5, *Pe* = 1.2); ❍ kraft pulp (kappa number = 17.6, *Pe* = 11.3); ❒ glass spheres (*Pe* = 64.6); – – – – plug flow; – · – · – perfectly mixed flow

groups of points. In contrast with sets I and II, the values of wash yield obtained in set of experiments III lie evidently in the region of small Peclet numbers. This discrepancy can be explained by the different properties of the pulp. While the pulp used in the first and second sets of experiments did not contain any apparent fibre knots, the visual observations of the pulp used in the third set of experiments confirmed the presence of undercooks in it.

The suitability of Eq. (*13*) used to determine the Peclet number was verified by comparing the theoretical with experimental washing curves (Fig. 3). The washing curve given by Brenner[8] for Pe = 8 as the dependence of the exit dimensionless concentration, c_e/c_0, on the dimensionless time, T, was plotted together with the experimental data obtained in the form of the dependence of c_e/c_0 *versus* the dimensionless time, Θ. The shape of theoretical washing curve (Pe = 8) was less steep in comparison with the

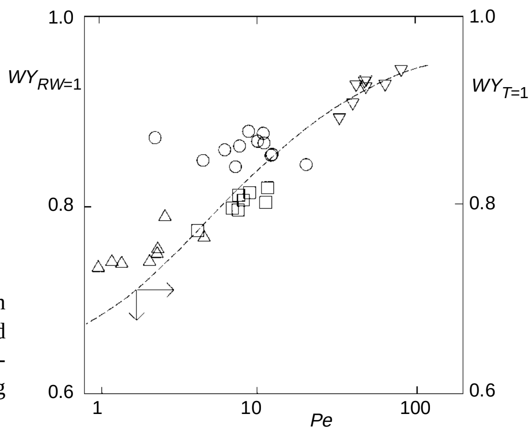

FIG. 2
Displacement wash yield $WY_{RW=1}$ as a function of Peclet number: ❍ set I (see Tables I and II); ❐ set II; Δ set III; ∇ set IV; – – – – theoretical dependence of $WY_{T=1}$ on Pe according to Brenner[8]

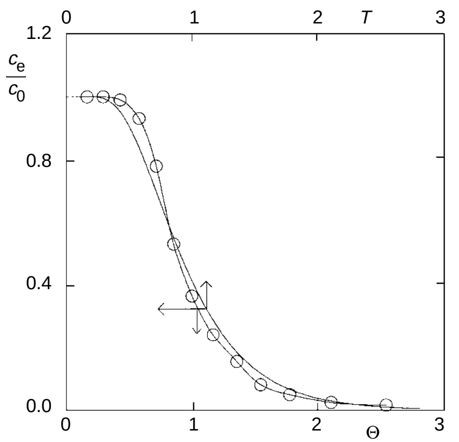

FIG. 3
Comparison of theoretical washing curve with experiments: —— theoretical dependence of c_e/c_0 on T according to Brenner[8] (Pe = 8); ❍ experimental points for Pe = 8.8 (dependence of c_e/c_0 on Θ)

experimental data (Pe = 8.8). In order to use the step function boundary conditions (see Eqs (*3*) and (*4*)), the input stream should instantaneously change from the solution initially saturating the bed to pure water. Although this ideal situation is never achieved, the actual change in concentration was assumed to be abrupt enough so that the exit concentration data might be treated using the way developed for the step change concentrations. This assumption may be valid for thick beds, however, for thin beds this assumption may be appreciably in error. Thus, the end effects could influence washing curves obtained experimentally. It has to be reminded that, in our case, the length ratio, h/d, was equal to 0.82 only. In contrast with thick packed beds which may be characterized as infinite in thickness, significant macroscopic variations in the flow conditions at different locations may be expected to occur in flow through thin packed beds of finite thickness. In this case, the lateral dispersion of wash liquid has not to be great enough to ensure a uniform wash liquid concentration at any given cross section. In spite of these facts, it is worth mentioning that, for glass spheres, the values of the wash yield, $WY_{RW=1}$, plotted as a function of the Peclet number calculated from Eq. (*13*) (Fig. 2), show a satisfactory correspondence with the dependence of the theoretical wash yield, $WY_{T=1}$, on the Peclet number according to Brenner[8].

As for the values of the wash yield obtained in our work, Trinh *et al.*[5] reported the wash yield, $WY_{RW=1}$, varying from 0.84 to 0.87 for initial lignin bed concentration of 25 kg m^{-3} under similar experimental conditions (temperature and superficial velocity of wash liquid, bed thickness and bed consistency). The values of wash yield reported by Trinh *et al.*[5] agree with those obtained in our work for initial bed lignin concentration of 29 kg m^{-3}.

On the basis of our own data measured for pulp bed, the following equation was derived for the quantitative evaluation of the effect of the dispersion wash liquid and initial lignin concentration, X_0, on the wash yield

$$WY_{RW=1} = 0.65\, Pe^{\,0.016}\, X_0^{-0.15} \quad . \qquad (25)$$

The suitability of Eq. (*25*) was judged on the basis of the mean relative quadratic deviation of the wash yield, δ (defined in Symbols), which was 1.8% (maximum deviation of 4.8%, n = 28). The values of the Peclet number and those of the initial lignin concentration, X_0, varied in the ranges of 1.0–20 and 0.19–0.44 g of lignin/g of oven dried pulp, respectively. Since the values of regression coefficients, which were evaluated by the least square method, represent an estimate of the real values, the 95% confidence intervals were also calculated. They are for the coefficient (0.256; 1.047), for the power of the Peclet number (0.0141; 0.0187), and for the power at the initial lignin concentration (–0.145; –0.157). Since the effect of the Peclet number on the wash yield in Eq. (*25*) is little marked, the former was omitted from the treatment to obtain

$$WY_{RW=1} = 0.65\, X_0^{-0.18} \qquad (26)$$

with a mean relative quadratic deviation δ = 2.0% (maximum deviation of 4.8%).

For comparison with the correlation given by Eq. (*25*), the theoretical displacement wash yield, $WY_{T=1}$, evaluated according to Brenner[8] was expressed as a function of the Peclet number only in the form

$$WY_{T=1} = 0.69\, Pe^{\,0.084}\ , \qquad (27)$$

where the Peclet number ranging from 0.8 to 24 where theoretical washing curves were evaluated in ref.[8].

Our experimental results obtained for the static bed of glass spheres provided the correlation between the wash yield and the Peclet number as

$$WY_{RW=1} = 0.75\, Pe^{\,0.052} \qquad (28)$$

with a mean relative quadratic deviation of the wash yield, δ, equal to 1.0% (maximum deviation of 1.4%, $n = 8$) in the range of the Peclet number from 33 to 82. A correlation between the theoretical wash yield, $WY_{T=1}$, (ref.[8]) and the Peclet number can be expressed in the form

$$WY_{T=1} = 0.78\, Pe^{\,0.043}\ , \qquad (29)$$

where the Peclet number varied within the limits from 24 to 80, *i.e.* Eq. (*29*) was derived for the values of the Peclet number given by Brenner[8]. From comparison of Eqs (*28*) and (*29*) one can find that, for glass spheres, the Peclet number played more important part in experiments than under theoretical conditions. However, Eqs (*25*) and (*27*)–(*29*) indicate that the wash yield depends on the Peclet number in a small degree. For the given sort of pulp, it can be expected that the wash yield depends mainly upon the lignin concentration in mother liquor, X_0, defined as the mass of lignin per unit mass of oven dried pulp.

Dispersion Coefficient

The dispersion of lignin in the pulp bed was characterized by a dimensionless parameter, the Peclet number containing the dispersion coefficient, *D*, which is analogous to and has the same units as the diffusion coefficient. According to Sherman[17], the dispersion coefficient in beds of granular and synthetic fibrous media is of the order of 10^{-6} m^2 s^{-1} for the displacement velocities normally encountered in practice, while the diffusion coefficient is of the order of 10^{-9} m^2 s^{-1} for the diffusion-controlled mixing at a planar interface between dilute aqueous solutions (ref.[4]). As varying flow conditions can be assumed in different parts of the pulp bed, an average value of the dispersion coefficient was found. It is obvious from Fig. 4 that the values of the dispersion coefficient evaluated from Eq. (*8*) show a large scattering due to very different structures of pulp bed formed, especially for the third set of experiments in which the pulp with undercooks was used. In spite of the scatter in the results, the dependence of the dispersion coefficient on the superficial wash liquid velocity shows an increasing trend. The presence of undercooks in pulp used in the third set of experiments led to an increase in the dispersion coefficient. It seems that the pulp type, *i.e.* fibre characteristics, was the main variable affecting the dispersion coefficient. Also, the difference in geometry, that is, in average pore size and in pore size distribution, occurring in fibre material influenced dispersion in the given pulp bed.

The influence of the superficial wash water velocity on the shape of the normalized washing curve is illustrated in Fig. 5 where data obtained for the first set of experiments (see Table II) are plotted. From Fig. 5 follows that the superficial velocity did not appear as a significant effect on the shape of washing curves. Also, Lee[3] found no significant effect of superficial velocity on the wash yield in the range from 7.2 . 10^{-5} to 7.5 . 10^{-4} m s^{-1}. Trinh *et al.*[5], on the other hand, reported that the wash yield was

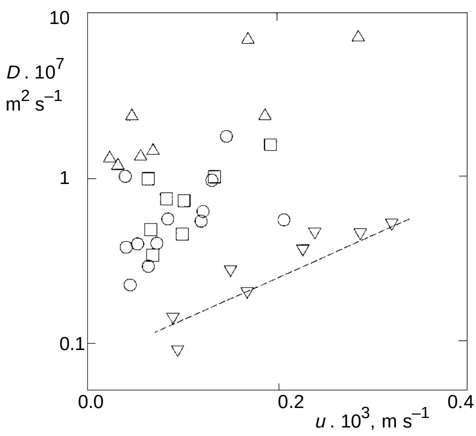

FIG. 4
Effect of superficial wash liquid velocity on dispersion coefficient: Symbols: see legend to Fig. 2; – – – – Eq. (*30*)

found to increase with superficial velocity at low bed consistency (3%) only, however, no effect was observed at high consistency (15%).

The data obtained for spherical particles result in a straight-line relationship between the dispersion coefficient, D (in $m^2\ s^{-1}$), and the superficial liquid velocity, u (in $m\ s^{-1}$), as follows

$$D = 1.63 \cdot 10^{-3} u \qquad (30)$$

with a correlation coefficient of 0.96. Equation (*30*) was derived for the superficial wash water velocity ranging from $8.9 \cdot 10^{-5}$ to $3.2 \cdot 10^{-4}$ m s^{-1}. The linear function between the dispersion coefficient and the superficial liquid velocity obtained for glass spheres is in accord with the results by Montillet *et al.*[18] who reported that the dispersion coefficient is a linear function of the superficial velocity for beds formed by spheres, Raschig rings, and nickel foams. For the superficial velocity ranging from $3 \cdot 10^{-4}$ to $1.3 \cdot 10^{-2}$ m s^{-1}, Sherman[17] found that the mixing parameter, D/u, was a constant for beds built up by either synthetic fibres (viscose, Dacron) or 3 mm glass spheres. Of course, in contrast with pulp fibres, the synthetic fibres had a constant diameter.

Mean Residence Time and Space Time

The space time, τ, which is defined as the void volume of the bed divided by the volumetric flow rate of wash liquid, can be expressed by the following equation

$$\tau = \frac{A \varepsilon h}{V} \qquad (31)$$

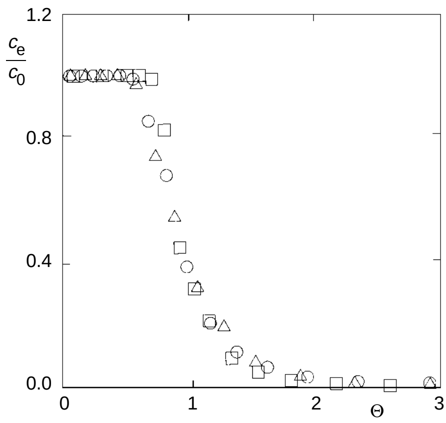

FIG. 5
Effect of superficial wash liquid velocity on output response to step change in concentration for kraft pulp: ❍ $u = 4.4 \cdot 10^{-5}$ m s^{-1}, $Pe = 11.0$; Δ $u = 1.2 \cdot 10^{-4}$ m s^{-1}, $Pe = 10.9$; ❐ $u = 2.1 \cdot 10^{-4}$ m s^{-1}, $Pe = 20.4$

and compared with the mean residence time, t_m, defined by Eq. (*10*). As follows from Fig. 6, the mean residence time of lignin was always longer than the space time for pulp bed while the mean residence time of lignin was approximately the same as the space time for bed of spherical particles. The correlations between the mean residence time and the space time were derived in the form

$$t_m = 1.68\,\tau \qquad (32)$$

with a correlation coefficient of 0.97 for fibres, and

$$t_m = 0.96\,\tau \qquad (33)$$

with a correlation coefficient of 0.98 for glass spheres.

The removal of lignin from pulp bed is accomplished *via* a combination of displacement and diffusion processes. If the solid particles in the bed are porous, then lignin may be initially contained both within the solid particles and in the solution contained in the interparticle voids. A part of lignin must diffuse out of the internal structure of the fibre to the external surface of the liquid layer immobilized on the external surface of the fibre before it can become available for displacement. Because of fibre swelling, the amount of mother liquor contained in the fibre lumens and walls can be considerable, depending upon the pulp consistency. Diffusion of lignin from the stagnant areas between fibres should be considered as well. The relationship between the mean residence time and the space time obtained for pulp bed confirmed that the displacement washing was not a purely mechanical process but diffusion must be taken into

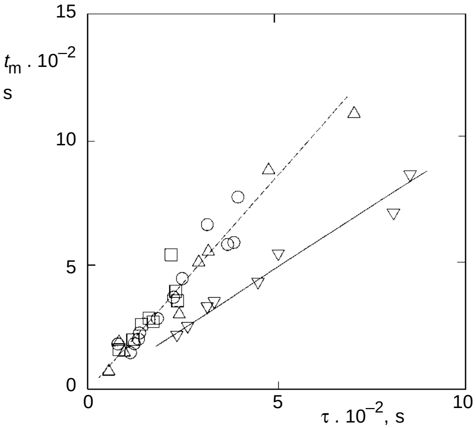

FIG. 6
Mean residence time as a function of space time: Symbols: see legend to Fig. 2; – – – – Eq. (*32*); ——— Eq. (*33*)

account in the case when the presence of mother liquor inside fibres cannot be neglected.

Bed Efficiency

Finally, the bed efficiency defined by Eq. (*23*) was evaluated at the wash liquor ratio equal to unity. In case of pure displacement, when the perfect plug flow exists, the efficiency increases from one to infinity with increasing wash liquor ratio from 0 to 1. For perfectly mixed vessel, when the outlet lignin concentration is equal to the average concentration inside the bed, the efficiency is equal to unity. Results obtained for the actual displacement washing at the wash liquor ratio equal to unity are shown in Fig. 7. For non-porous glass spheres, when displacement prevails over dispersion at the interface between the wash liquid and the entrained liquor, the bed efficiency is greater than for the pulp bed in which both displacement and diffusion are operating simultaneously.

The dependence of the bed efficiency upon the Peclet number and initial bed lignin concentration, X_0, may be described by the following correlation

$$E_{RW=1} = 0.95\,Pe^{\,0.031}\,X_0^{\,-0.41} \qquad (34)$$

with a mean relative quadratic deviation of bed efficiency, δ, equal to 4.6% (maximum deviation of 11.0%, n = 28). The correlation (*34*) was derived over the same ranges of the Peclet number and initial bed lignin concentration as Eq. (*25*). The 95% confidence intervals of regression coefficients in Eq. (*34*) were evaluated. They are for the coefficient (0.547; 1.359), for the power of the Peclet number (0.0244; 0.0368), and for the

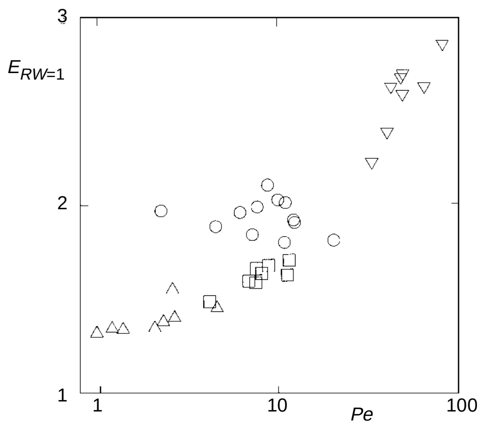

FIG. 7
Bed efficiency at RW = 1 as a function of Peclet number: Symbols: see legend to Fig. 2

power of the initial lignin concentration (–0.395; –0.427). Since the influence of the Peclet number in Eq. (*34*) on the value of bed efficiency is not marked, the following correlation in the approximate form

$$E_{RW=1} = 0.94\, X_0^{-0.46} \tag{35}$$

was expressed (a mean relative quadratic deviation δ = 5.1%, maximum deviation of 11.1%). For pulp beds, the Peclet number slightly influenced the bed efficiency similarly as in case of the wash yield (see Eq. (*25*)). However, the initial bed lignin concentration had a significant effect on the bed efficiency.

Furthermore, a correlation between the bed efficiency and the Peclet number was derived for glass sphere bed in the form

$$E_{RW=1} = 1.05\, Pe^{\,0.23} \tag{36}$$

with a mean relative quadratic deviation of bed efficiency δ = 4.0% (maximum deviation of 5.8%, n = 8). In comparison with pulp bed, a more expressive influence of the Peclet number on the bed efficiency appears for glass sphere bed in which lignin diffusion from particles to the wash liquid does not exist.

CONCLUSIONS

At the dimensionless length ratio h/d = 0.82, displacement washing of kraft pulp was simulated and the results obtained showed several differences between a bed of non-porous, non-compressible glass spheres and a bed of the more complex fibrous system. Because of a much wider pore size distribution and fibre swelling, the wash yield as well as the bed efficiency are lower for pulp bed in comparison with bed of glass spheres. Displacement of mother liquor from pulp bed was evidently accompanied by diffusion process. In spite of small diffusion rate, the contribution of diffusion to the washing effectiveness can be distinguished in cases of sufficiently long contact time between fibres and wash liquid.

SYMBOLS

A	cross-sectional area of pulp bed, m^2
a	specific surface of fibre material, m^{-1}
B	permeability coefficient, m^2
C	consistency (*i.e.* concentration of moisture-free pulp given as mass of fibres per unit volume of uniform bed), kg m^{-3}
C_i	dimensionless concentration in Eq. (*7*) [= $(c_i - c_\mathrm{V})/(c_0 - c_\mathrm{V})$]

c_i	local solute (in our case lignin) concentration in Eq. (*2*), kg m^{-3}
c_e	exit solute (in our case lignin) concentration from bed, kg m^{-3}
c_L	average lignin concentration inside bed, kg m^{-3}
c_V	solute (in our case lignin) concentration in wash liquor, kg m^{-3}
c_0	initial solute (in our case lignin) concentration in bed at $t = 0$, kg m^{-3}
D	dispersion coefficient, m^2 s^{-1}
DR	displacement ratio defined by Eq. (*22*)
d	inside diameter of washing cell, m
E	bed efficiency defined by Eq. (*20*)
E_c	frequency distribution of dimensionless residence time defined by Eq. (*12*)
$E_{RW=1}$	bed efficiency at $RW = 1$ defined by Eq. (*23*)
h	thickness of bed, m
K	Kozeny constant (dimensionless) in Eq. (*15*)
l_n	numerical average length of fibres, mm
l_w	weighted average length of fibres, mm
n	number of measurements
Δp	pressure drop, Pa
Pe	Peclet number (= $hu/D\varepsilon$)
R	hydraulic mean radius of pores defined by Eq. (*16*), m
RW	wash liquor ratio
T	dimensionless time defined by Eq. (*6*)
t	time from start of experiment, s
t_m	mean residence time defined by Eq. (*10*), s
u	superficial wash liquid velocity, m s^{-1}
V	volumetric flow rate of wash liquid, m^3 s^{-1}
v	effective specific volume of fibre material, m^3 kg^{-1}
$WY_{RW=1}$	wash yield at $RW = 1$ defined by Eq. (*1*)
$WY_{T=1}$	wash yield at $T = 1$ defined by Eq. (*24*)
X_0	initial bed lignin concentration in Eqs (*25*), (*26*), (*34*), and (*35*), g of lignin/g of oven dried pulp
z	length coordinate in Eq. (*2*), m
Z	dimensionless length coordinate in Eq. (*7*) (= z/h)
δ	mean relative quadratic deviation of wash yield (or also bed efficiency) defined as $$\delta = \left[\frac{1}{n} \sum_{1}^{n} \left(\frac{WY_{exp} - WY_{calc}}{WY_{exp}} \right)^2 \right]^{1/2} . 100 \ , \ \%$$
ε	average porosity of pulp bed
Θ	dimensionless time defined by Eq. (*9*)
μ	viscosity of wash liquid, Pa s
σ^2	variance defined by Eq. (*11*)
τ	space time defined by Eq. (*31*), s

Subscripts

calc	referring to value of wash yield calculated from Eqs (*25*), (*26*), and (*28*), or to value of bed efficiency calculated from Eqs (*34*)–(*36*)
exp	referring to experimental value

REFERENCES

1. Crotogino R. H., Poirier N. A., Trinh D. T.: Tappi *70*, 95 (1987).
2. Grähs L. E.: Sven. Papperstidn. *78*, 446 (1975).
3. Lee P. F.: Tappi *62*, 75 (1979).
4. Lee P. F.: Tappi *67*, 100 (1984).
5. Trinh D. T., Poirier N. A., Crotogino R. H., Douglas W. J. M.: J. Pulp. Paper Sci. *15*, J28 (1989).
6. Levenspiel O.: *Teorie a vypocty chemickych reaktoru*. SNTL, Praha 1967.
7. Luthi O.: Pulp Paper Can. *86*, T201 (1975).
8. Brenner H.: Chem. Eng. Sci. *17*, 229 (1962).
9. Test method T227 in: *Tappi Test Methods*. Tappi Press Atlanta, Georgia 1992.
10. Ref.[9]; Test method T236.
11. Ref.[9]; Test method T232.
12. Bischoff K. B., Levenspiel O.: Chem. Eng. Sci. *17*, 245 (1962).
13. Levenspiel O., Smith W. K.: Chem. Eng. Sci. *6*, 227 (1957).
14. van der Laan E. T.: Chem. Eng. Sci. *7*, 187 (1957).
15. Ingmanson W. L.: Chem. Eng. Prog. *49*, 577 (1953).
16. Cullinan H. T.: Appita J. *44*, 91 (1991).
17. Sherman W. R.: AIChE J. *10*, 855 (1962).
18. Montillet A., Comiti J., Legrand J.: Chem. Eng. J. *52*, 63 (1993).